Validity of the Long-Wavelength Approximation for Long Arm 3rd Generation Interferometers

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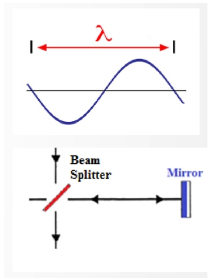
Purpose:

To analyze the response of the long arm, 3rd generation interferometer to realistic GW forms from CCSNe.

•The long-wavelength approximation assumes that as a photon travels along an arm, it will experience a change in the spacetime strain, or approximately, it will see only one particular phase during its total trip time.

•When the length of the LIGO arms are comparable to the gravitational wavelength, as is the case with high GW frequencies (kHz range)



Illustrating Example: If the time the photon spends in an arm is identical to the period of the GW, then there will be no phase change, and the interferometer will be insensitive to the GW.

The Response Function

• Phase difference for the light, single photon along one LIGO arm (frequency domain)

$$\delta\widetilde{\psi}(f_{GW}) = -\frac{2\pi}{c} f\,\widetilde{h}_{xx}(f_{GW})\ D(f_{GW}, n_x)$$

$$D(f_{GW}, n_x) = \frac{c}{i4\pi L\, f_{GW}}\left[\frac{1-e^{-i(1-n_x)2\pi f_{GW}\frac{L}{c}}}{1-n_x} - e^{-i4\pi f_{GW}\frac{L}{c}}\left(\frac{1-e^{i(1+n_x)2\pi f_{GW}\frac{L}{c}}}{1+n_x}\right)\right]$$

Rakhmanov et. al., 2008 *Classical Quantum Gravity* **25** 184017

•D is the response function

•c – speed of light; L- unperturbed LIGO arm length

•T- unperturbed travel time, one way

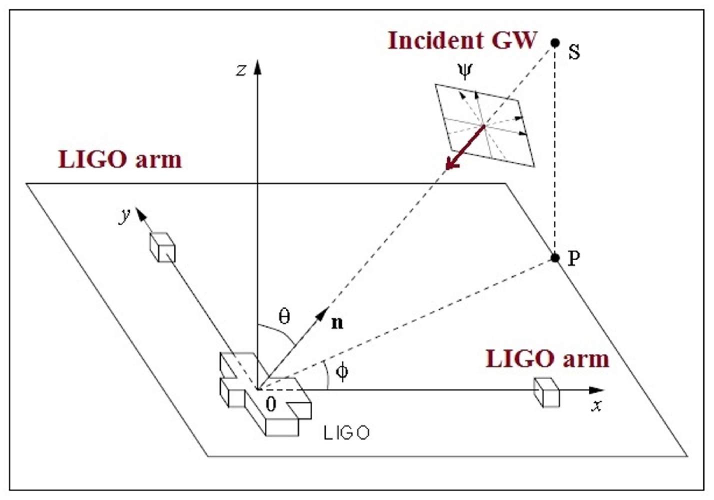
$$T = \frac{L}{c}$$

•Orientation of GW with respect to LIGO arms

$n_x = \sin\theta\cos\phi,\ n_y = \sin\theta\sin\phi,\ n_z = \cos\theta$

•The gravitational wave along the LIGO x-arm

$$h_{xx}^{TT} \propto \ddot{I}_{xx}$$



•The *long wavelength approximation* is the limiting behavior of the transfer function when f → 0, and in this case, D=1.

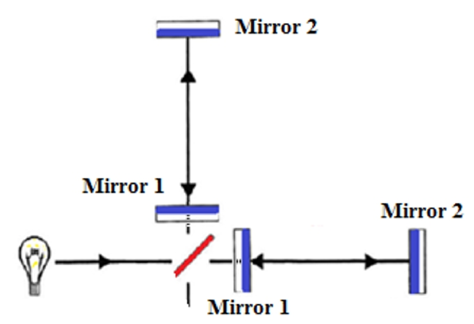
• Fabry-Perot Cavities

•In addition, there are Fabry-Perot cavities situated along each LIGO arm. They effectively extend the arm length by reflecting the laser several hundred times before passing it to the out port.

$$H(f_{GW}, n_x) = C(f_{GW})\, D(f_{GW}, n_x)$$

$$C(f) = e^{i2\pi f_{GW} T}\frac{\sinh(2\pi f_0 T)}{\sinh\left[2\pi f_0 T\left(1+\frac{i f_{GW}}{f_0}\right)\right]}$$

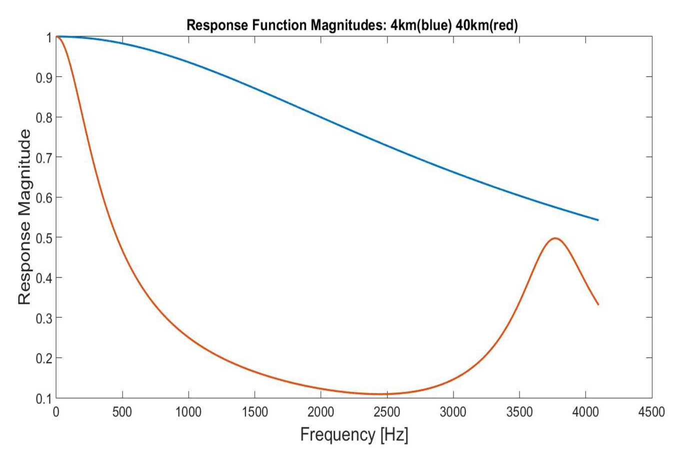
•Where $f_0 = \frac{|\ln(r_1 r_2)|}{4\pi T}$ is the lowest pole order, and r1 and r2 are the reflectivity of the mirrors.

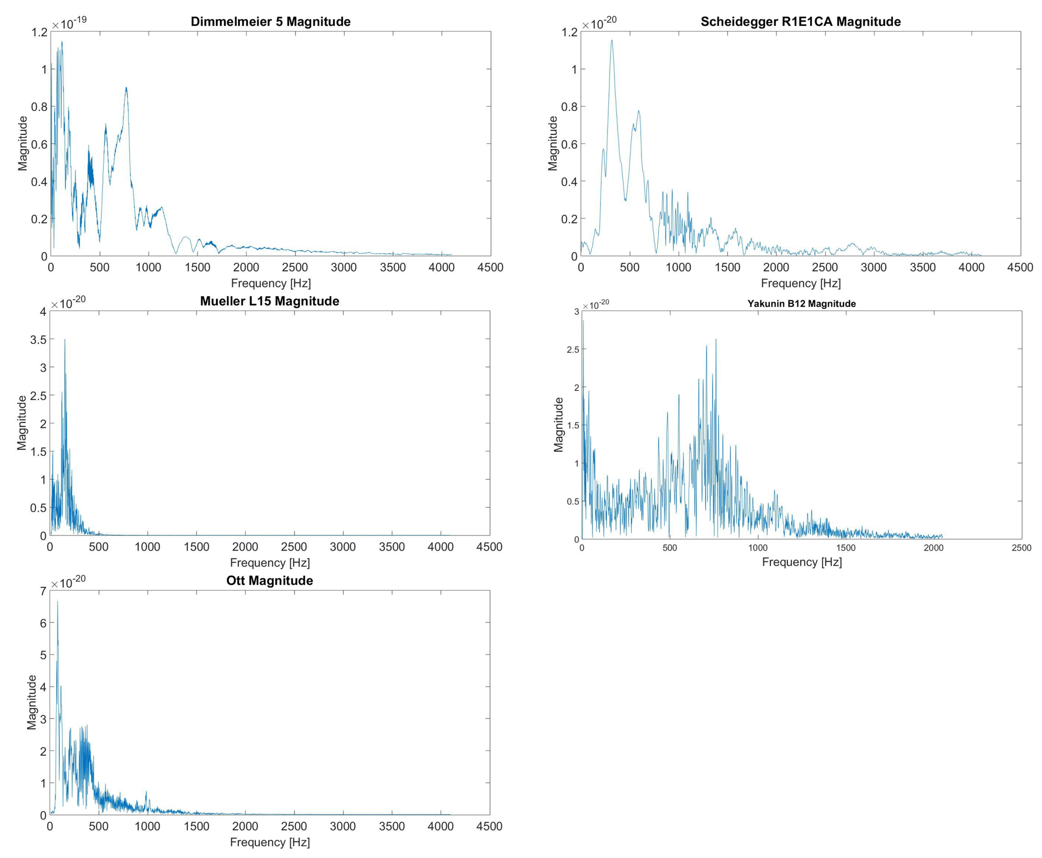


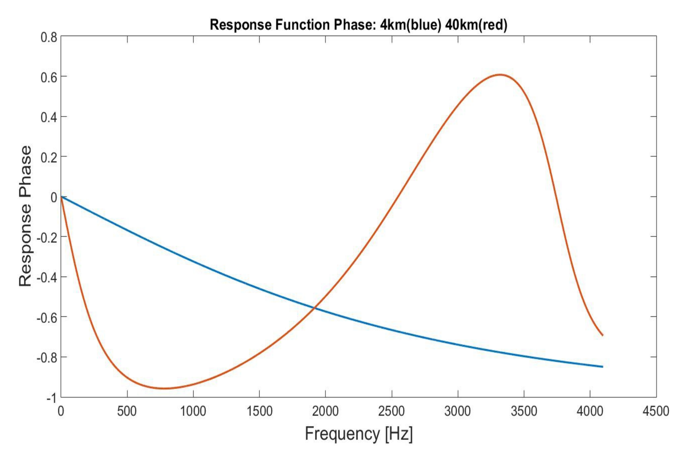
Results

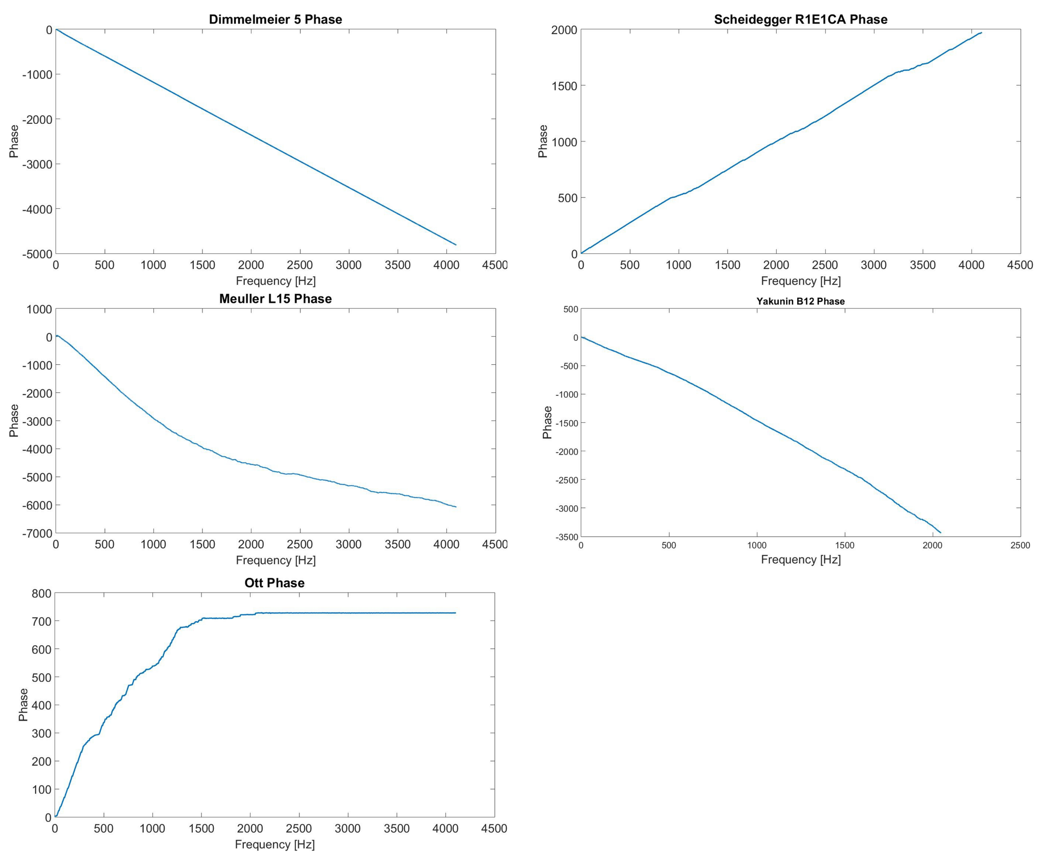
Magnitude and Phase for the Response Function and Wave Forms, Frequency Domain

- The 4km (blue) phases and magnitudes are plotted against the 40km (red/brown)
- At around 1kHz, more than 70% of the signal is lost with the Response Magnitude plot



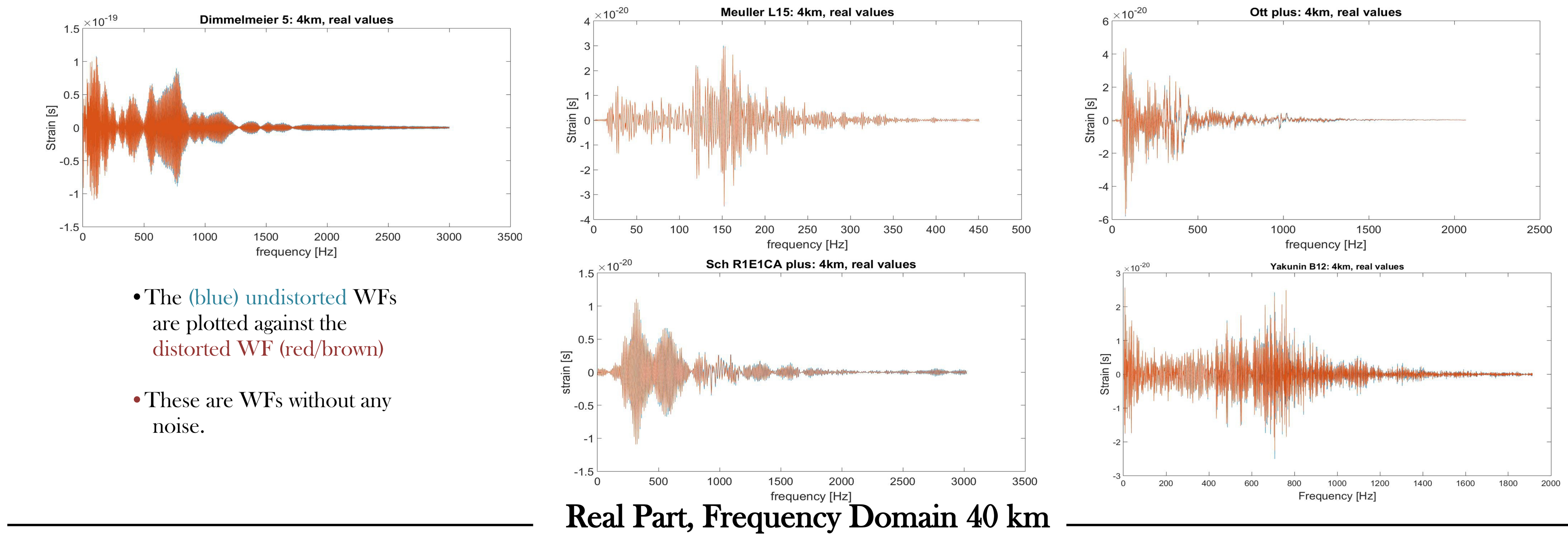






Wave Forms

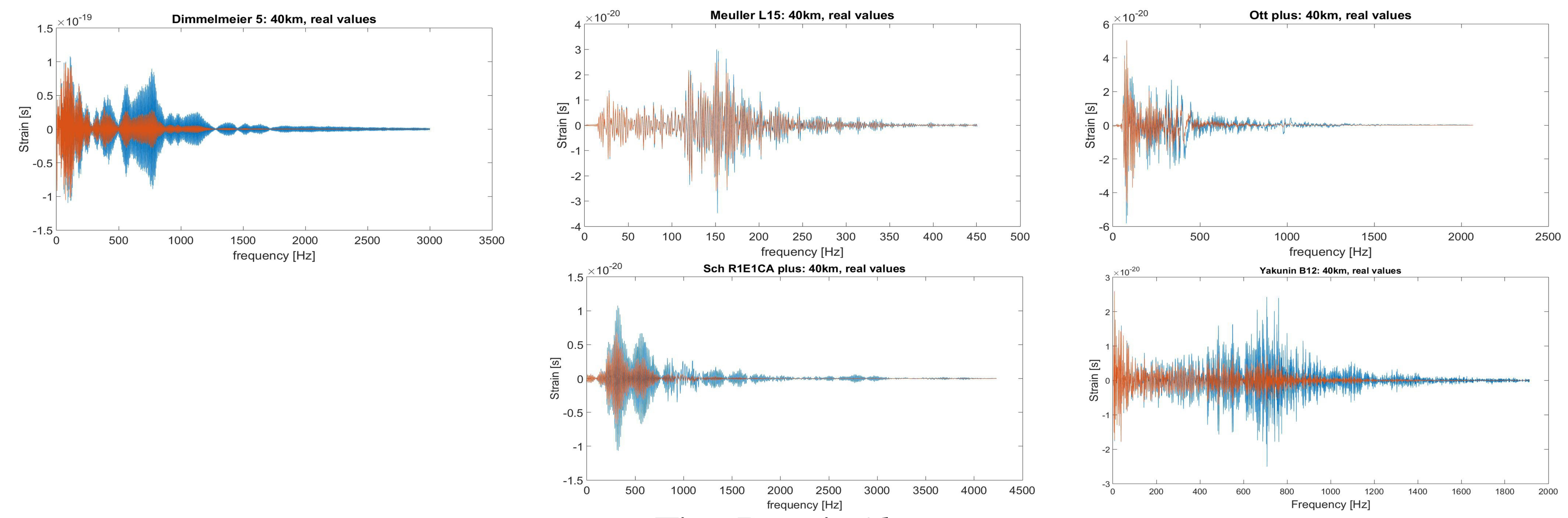
Real Part, Frequency Domain 4 km



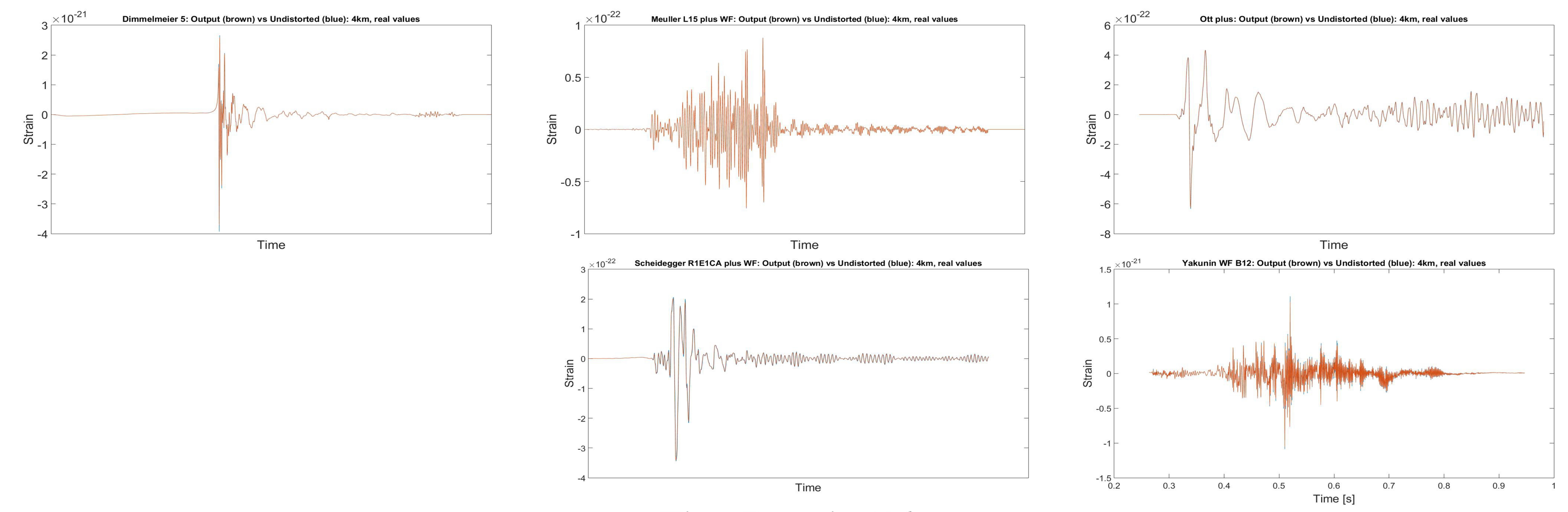
•The (blue) undistorted WFs are plotted against the distorted WF (red/brown)

•These are WFs without any noise.

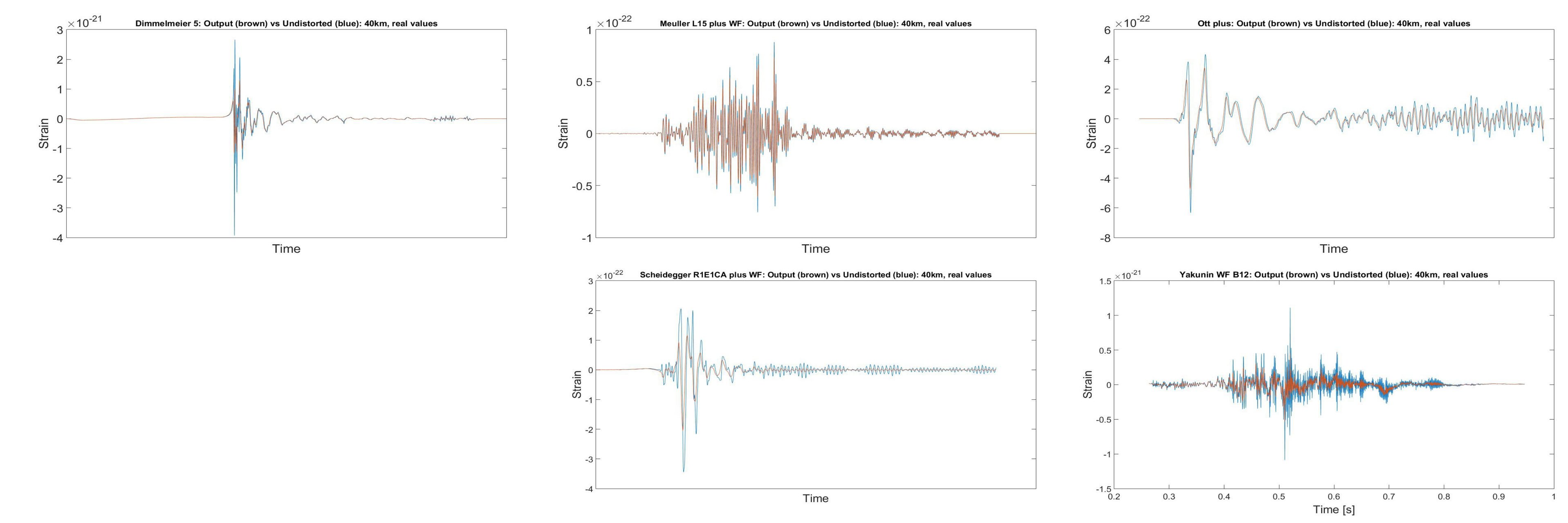
Real Part, Frequency Domain 40 km



Time Domain 4 km



Time Domain 40 km



Summary:

- The study of the impact of the transfer function seems to indicate that the response of the interferometer at frequencies above a few hundred Hertz needs to be carefully evaluated for High frequencies.
- The next step of this analysis is to quantify how the calibration process is affected by the low-response function at high frequencies for the 40 km interferometer.

Project Wiki Page: https://wiki.ligo.org/viewauth/Bursts/LWApproximations